Saptarshi

Measurements

Q. Define unit, give requirements of good units.

The reference standard used for the measurement of a physical quantity is called a unit.

A good unit should have the following properties:

i) It should be easily available.

ii) It should be invariable (should not change in space and time).

iii) It should be universally accepted.

iv) It should be reproducible and not perishable.

Q.Explain the terms fundamental units and derived units.

- **Fundamental quantities:** the physical quantities which do not depend on any other physical quantities for their measurements are known as fundamental quantities.

  Fundamental units: The units used to measure fundamental quantities are called fundamental units.

- **Derived quantities and derived units:**

  Physical quantities other than fundamental quantities which depend on one or more fundamental quantities for their measurements are called derived quantities, e.g. force, speed etc. the unit of derived quantities are called derived units.

  e. g. newton, kg/m^3, etc. the table shows a list of derived quantities with their units and symbols along with dimensions.

SI System: SI system is international system of units. It consists of seven fundamental units, two supplementary units and large number of derived units.

Fundamental quantities with S. I. units and symbols

	Fundamental quantities	S. I. Units	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	Kelvin	K
5.	Electric current	Ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

Supplementary units

1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

Some conventions to be followed while writing S.I. units are given below.

i) Full name of unit always start with small letter even if named after a person. e.g. newton, joule and not Newton, Joule.

ii) Symbol for unit named after a person should be in capital letters e.g. 'N' for newton, 'J' for joule, 'A' for ampere etc.

iii) Symbols for all other units are written in small letters e.g. 'm' for meter, 's' for second etc.

iv) Symbols of units are not be expressed in plural form e. g. 25m and not 25 ms

v) Full stop and any other punctuation mark should not be written after the symbols e.g. kg and not kg. or N and not N.

Define dimensions

The dimensions of a physical quantity are the powers to which fundamental units must be raised in order to obtain the unit of that physical quantity.

For determining the dimensions of a physical quantity, the units of the fundamental quantities are represented by 'L' for length, 'M' for mass, 'T' for time, 'K' for temperature,' I' for current, 'C' for luminous intensity and 'mol ' for amount of substance.

An expression, which gives the relation between the derived units and fundamental units in the term of dimensions, is called a dimensional equation.

Obtain dimensions of following quantities.

1. **Mass** = $\left[L^0\ M^1\ T^0\right]=\left[M^1\right]$
2. **Length** = $\left[L^1\ M^0\ T^0\right]=\left[L^1\right]$
3. **Time** = $\left[L^0\ M^0\ T^1\right]=\left[T^1\right]$
4. **Electric Current** = $\left[L^0\ M^0\ T^0\ I^1\right]=\left[I^1\right]$
5. **Temperature** = $\left[L^0\ M^0\ T^0\ K^1\right]=\left[K^1\right]$

   Note that Mechanics deals with only three fundamental quantities length, mass and time. Hence, dimensions of quantities in mechanics consist of only length, mass and time.

6. **Displacement** = $\left[L^1\ M^0\ T^0\right]$
7. **speed**

$$\therefore \text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\left[L^1\ M^0\ T^0\right]}{\left[T^1\right]} = \left[L^1\ M^0\ T^{-1}\right]$$

8. Velocity

$$\text{velocity} = \frac{\text{Displacement}}{\text{time}} = \frac{\left[L^1\ M^0\ T^0\right]}{\left[T^1\right]} = \left[L^1\ M^0\ T^{-1}\right]$$

9. Acceleration

$$\left[\text{Acc}^n\right] = \frac{\text{Change in velocity}}{\text{time}} = \frac{\left[L^1\ M^0\ T^{-1}\right]}{\left[T^1\right]} = \left[L^1\ M^0\ T^{-2}\right]$$

10. Force

$$\text{Force} = \text{mass}\ \left[\text{acc}^n\right] = \left[L^0\ M^1\ T^0\right]\left[L^1\ M^0\ T^{-2}\right] = \left[L^1\ M^1\ T^{-2}\right]$$

11. Work

$$\text{Work} = \text{force}\ \ \text{displacement} = \left[L^1\ M^1\ T^{-2}\right]\left[L^1\ M^0\ T^0\right] = \left[L^2\ M^1\ T^{-2}\right]$$

12. Power

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\left[L^2\ M^1\ T^{-2}\right]}{\left[T^1\right]} = \left[L^2\ M^1\ T^{-3}\right]$$

13. Pressure

$$\text{Pressure} = \frac{\text{force}}{\text{area}} = \frac{\left[L^1\ M^1\ T^{-2}\right]}{\left[L^2\right]} = \left[L^{-1}\ M^1\ T^{-2}\right]$$

14. Impulse

$$\text{Impulse} = \text{Force}\ \ \text{time} = \left[L^1\ M^1\ T^{-2}\right]\left[L^0\ M^0\ T^1\right] = \left[L^1\ M^1\ T^{-1}\right]$$

15. Electric Charge

$$\text{charge} = \text{current} \times \text{time} = \left[L^0\ M^0\ T^1\ I^1\right]$$

16. Potential Difference

$$\text{P.D.} = \frac{\text{Work}}{\text{Charge}} = \frac{\left[L^2\ M^1\ T^{-2}\ I^0\right]}{\left[L^0\ M^0\ T^1\ I^1\right]} = \left[L^2\ M^1\ T^{-3}\ I^{-1}\right]$$

17. Temperature gradient

$$\text{Temperature gradiant} = \frac{\text{temperature}}{\text{distance}} = \frac{\left[K^1\right]}{\left[L^1\right]} = \left[K^1\ L^{-1}\right]$$

. Dimensions of various physical quantities in mechanics are given in table below:

Sr.No.	Derived quantity	Formula	Dimensions	S.I. unit	Symbol
1.	Area	$A = L^2$	$\left[L^2\ M^0\ T^0\right]$	Square meter	m^2

2.	Volume	$V = L^3$	$\left[L^3\ M^0\ T^0\right]$	Cubic meter	m^3
3.	Density	$d = M/V$	$\left[L^{-3}\ M^1\ T^0\right]$	kilogram per cubic meter	Kg/m^3
4.	Velocity	$v = s/t$ (speed)	$\left[L^1\ M^0\ T^{-1}\right]$	meter per second	m/s
5.	Acceleration	$a = v/t$	$\left[L^1\ M^0\ T^{-2}\right]$	meter per second square	m/s^2
6.	Momentum	$P = m/v$	$\left[L^1\ M^1\ T^{-1}\right]$	kilogram meter per second	kg m/s
7.	Force	$F = ma$	$\left[L^1\ M^1\ T^{-2}\right]$	newton	N
8.	Impulse	$J = Ft$	$\left[L^1\ M^1\ T^{-1}\right]$	newton second	Ns
9.	Work	$W = Fs$	$\left[L^2\ M^1\ T^{-2}\right]$	joule	J
10.	Kinetic Energy	$K.E. = \frac{1}{2}mv^2$	$\left[L^2\ M^1\ T^{-2}\right]$	joule	J
11.	Potential Energy	$P.E. = mgh$	$\left[L^2\ M^1\ T^{-2}\right]$	joule	J
12.	Power	$P = \frac{w}{t}$	$\left[L^2\ M^1\ T^{-3}\right]$	joule per second or watt	J/s or W
13.	Pressure	$P = \frac{F}{A}$	$\left[L^{-1}\ M^1\ T^{-2}\right]$	newton per square meter	N/m^2

Uses of Dimensional Analysis :

1) To find the correctness of physical equation.

The dimensions of all the terms on two sides of a physical equation must be same. This is called the principle of homogeneity of dimensions.

e.g. consider the equation $S = ut + \frac{1}{2}at^2$

By writing the dimensions, we get

$S = \left[L^1M^0T^0\right]$, $ut = \left[L^1M^0T^0\right]$, $at^2 = \left[L^1M^0T^0\right]$

The number $\frac{1}{2}$ has no dimensions. Thus the equation is dimensionally correct.

2) To find conversion factor between the units of the same physical quantity in two different systems of units.

e.g. to find the conversion factor between the units of the Force. i.e. newton in S.I. system to dyne in c.g.s. system.

Let 1 newton = $\times$ dyne……………….(1.1)

The dimensions of force are $\left[L^1M^1T^{-2}\right]$

$\therefore$ Equation (1.1) in dimension form can be written as

$$\left[L_2{}^1M_2{}^1T_2{}^{-2}\right] = \left[L_2{}^1M_2{}^1T_2{}^{-2}\right]$$

$$\therefore x = \frac{\left[L_1{}^1M_1{}^1T_1{}^{-2}\right]}{\left[L_2{}^1M_2{}^1T_2{}^{-2}\right]}$$

$$= \left(\frac{L_1}{L_2}\right)^1\left(\frac{M_1}{M_2}\right)^1\left(\frac{T_1}{T_2}\right)^{-2} \ldots\ldots\ldots\ldots(1.2)$$

Where suffix 1 indicates SI system and suffix 2 indicates C.G.S. system.

Writing units

$$x = \left(\frac{m}{cm}\right)^1\left(\frac{kg}{g}\right)^1\left(\frac{s}{s}\right)^{-2}$$

$$= \left(10^2\frac{cm}{cm}\right)^1\left(10^3\frac{g}{g}\right)^1 \; 1^{-2}$$

$$\therefore x = 10^5 \qquad \therefore 1 \text{ newton } = 10^5 \text{dyne}$$

---

3) To establish relationship between related physical quantities.

e. g. the period ‘T’ of oscillation of a simple pendulum depends on length ‘*l*’ and acceleration due to gravity ‘g’. Let us derive the relation between T, *l* and g.

Let us assume that $T = Kl^x g^y$ ……………………….(1)

K= constant which is dimensionless.

The dimensions of $T = \left[L^0M^0T^1\right]$

The dimensions of $l = \left[L^1M^0T^0\right]$

And the dimensions of $g = \left[L^1M^0T^{-2}\right]$

$\therefore$ Equation (1.3) can be dimensionally written as

$$\left[L^0M^0T^1\right] = K\left[L^1M^0T^0\right]^x\left[L^1M^0T^{-2}\right]^y$$

$$\therefore \left[L^0M^0T\right] = K\left[L^{x+y}M^0T^{-2y}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

By comparing the powers of L, M, T on both sides of equation, we get

0 = x+y and 1= -2y $\qquad \therefore y = -\frac{1}{2} \text{ and } x = +\frac{1}{2}$

Substituting these values of x and y in equation (1) we get

$$T = K\, l^{1/2} g^{-1/2} \qquad \text{i.e.} T = K\sqrt{\frac{l}{g}}$$

The value of 'K', which is numerical constant, cannot be found by using dimensional analysis and can be found out experimentally.

Order of magnitude and significant figures:

Order of magnitude of a physical quantity is defined as the value of its magnitude rounded off to the nearest integral power of 10.

The magnitude of any physical quantity can be expressed as $A \times 10^n$ where 'A' is a number such that $0.5 \leq A < 5$ and 'n' is an integer, called order of magnitude.

eg..

i) radius of earth = 6400 km = 0.64×10^7

Order of magnitude of radius of earth is 10^7

ii) magnitude of the charge of electron 1.6×10^{-19} C

Order of magnitude of charge of electron = 10^{-19}

iii) Length of a rod is 5 m = 0.5×10^1 m.

Order of magnitude of length of rod = 10^1 m.

<u>Significant figures</u>: it can be defined as a figure, which is of some significance, but it does not necessarily denote a certainty.

Rules for determining significant figures:

i) Retain only one uncertain digit in the measurement of a physical quantity.

ii) When the value of the measurement should be rounded of to be given number of significant figures, then the figures to be dropped is (1) less than 5, then the last significant figure is unchanged (2) 5 or greater than 5, the last significant figure is increased by one.

iii) The zeros on the right hand side of the number are significant because they indicate the accuracy of the instrument used for measurement.

iv) The zeros on the left hand side of the number are significant e.g. the number of 0.0753 has only three significant figures.

v) If the number of digits more than the number of significant figures, the number should be expressed in the power of 10 e.g. the mass of the earth is written as 5.98×10^{24} kg, as it is known only up to three significant figures.

Rounding Off

1) 17.368 expressed as 17.37 correct to 4 significant digits. In the given number, the 5th digit is greater than 5. When we delete the fifth digit, we add 1 to the 4th digit.

2) 17.361 expressed as 17.36 correct to 4 significant digits. The number beyond the 4th digit is less than 5. We omit it.

When number to be omitted is five then

3) 17.3652 is expressed as 17.37 correct to 4 significant digits, **Rule:** as 5th digit is 5 and there is non zero number after it hence add 1 to 4th digit.

4) 17.335 expressed as 17.34 **Rule:** as 5^{th} digit is 5 and digit preceding 5 is odd hence 1 is added to 4^{th} digit
5) 17.345 expressed as 17.34 **Rule:** if the digit preceding 5 is even, We omit it.

Types of Errors:

1) **Instrumental (or constant) error :**
   - These errors are caused due to faulty construction of instruments.
   - e.g. if a thermometer is not graduated properly i. e. one degree on the thermometer actually corresponds to 0.99^0 the temperature measured by such a thermometer will differ from its value by a constant amount. Hence, it is also called as constant error.
   - Such errors can be minimized by taking the same measurement with different accurate instruments.
2) **Systematic error (Persistent errors) :**
   - This is an error due to defective setting of an instrument.
   - If the pointer of an ammeter is not pivoted exactly at the zero of the scale, it will not point to zero when no current is passing through it.
   - Such errors can be minimized by detecting its causes.
3) **Personal errors:**
   - These errors are introduced due to fault of an observer taking readings, referred to as human errors.
   - They vary from person to person.
   - e.g. error due to non-removal of parallax between pointer and its image in case of a magnetic compass needle .
4) **Random error (Accidental):** Even after minimizing above types of errors, errors may occur due to different factors like change in temperature, pressure or fluctuation in voltage while the experiment is being performed. Such errors cannot be eliminated but can be minimized.

   The effect of errors can be minimized by :
   1) Taking a large magnitude of the quantity to be measured.
   2) Taking large number of readings and calculating their mean value.
   3) Using an instrument whose least count is as small as possible.

<u>Measurement of errors</u>

- **<u>Error</u>**: The difference between the true value and measured value of a physical quantity is called error, which may be positive or negative. The true value of a physical quantity can never be known. When we take sufficient large number of readings and find their mean, the mean is referred to as the mean value.
- **<u>Mean Value</u>** Suppose that 'n' readings taken for the measurement of a physical quantity are $a_1,\ a_2, ...a_n$ then the mean value is $a_{mean} = \frac{a_1 + a_2 + ...a_n}{n}$

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

- **Absolute error** : The magnitude of the difference between mean value and each individual value is called absolute error.

  Thus for measurement 'a_1' the absolute error is $|\Delta a_1| = |a_m - a_1|$

  Similarly in the measurement 'a_2' it is $|\Delta a_2| = |a_m - a_2|$ and so on.

- **Mean absolute error**: The arithmetic mean of all the absolute errors is called mean absolute error in the measurement of the physical quantity.

$$|\Delta a_m| = \frac{|\Delta a_1| + |\Delta a_2| + \ldots\ldots.. |\Delta a_n|}{n} \qquad = |\Delta a_m| = \frac{1}{n}\sum_{i=1}^{n} \Delta a_i$$

- **Relative error:** The ratio of the mean absolute error in the measurement of a physical quantity to its mean value is called relative error.

$$\therefore \text{Relative error} = \frac{|\Delta a_m|}{a_m}$$

- **Percentage error:** The relative error multiplied by 100 is called the percentage error.

$$\therefore \text{Percentage error} = \frac{|\Delta a_m|}{a_m} \times 100\%$$

Errors in compound physical quantities

a) error in $x \pm y = \text{absolute error in } x + \text{absolute error in } y$

$$= |\Delta x| + |\Delta y|$$

b) % error in xy or $\frac{x}{y} = \%\text{error in } x + \%\text{error in } y$

$$= \frac{|\Delta x|}{x} \times 100 + \frac{|\Delta y|}{y} \times 100$$

c) % error in $x^n = n \times \%\text{error in } x$

$$= n \times \frac{|\Delta x|}{x} \times 100$$

d) % error in $x^n y^m = n \times \%\text{error in } x + m \times \%\text{error in } y$

$$= n \times \frac{|\Delta x|}{x} \times 100 + m \times \frac{|\Delta y|}{y} \times 100$$

Solved Problems

1) **Find dimensions of gravitational constant (G) and Permittivity of vacuum (ε_0)**

Ans :

(a) According to Newton's law of gravitation , force of attraction between two masses is given by , (from 12th std syllabus)

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = \frac{F\left[r^2\right]}{m_1 \quad m_2}$$

$$G = \frac{\left[M^1\ L^1\ T^{-2}\right]\left[L^2\right]}{\left[M^1\right]\left[M^1\right]}$$

$$G = \left[M^{-1}\ L^3\ T^{-2}\right]$$

(b) According to coulomb's law of electrostatics (to be learned in cha 10) force of attraction or repulsion between two charges is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$\varepsilon_0 = \frac{1}{4\pi F}\frac{q_1 q_2}{r^2}$$

$$\varepsilon_0 = \frac{q_1 \quad q_2}{F\left[r^2\right]}$$

$$\varepsilon_0 = \frac{\left[I^1T^1\right]\left[I^1T^1\right]}{\left[L^1\ M^1\ T^{-2}\right]\left[L^2\right]}$$

$$\varepsilon_0 = \left[L^{-3}\ M^{-1}\ T^4\ I^2\right]$$

---

2) Force acting on charge q moving with velocity v in magnetic field 'B' is given by F = qvB , find dimensions of magnetic field.

F = qvB

$$B = \frac{F}{qv}$$

$$B = \frac{\left[L^1\ M^1\ T^{-2}\right]}{\left[I^1\ T^1\right]\left[L^1\ T^{-1}\right]}$$

$$B = \left[L^0\ M^1\ T^{-2}\ I^{-1}\right]$$

---

3) Show that 1J = 10^7 erg

Let $1J = x\ \text{erg}$

The dimensions of work are $\left[L^2\ M^1\ T^{-2}\right]$

$\therefore$ Equation in dimension form can be written as

$$\left[L_2^{\,2}M_2^{\,1}T_2^{\,-2}\right]=\left[L_2^{\,2}M_2^{\,1}T_2^{\,-2}\right]$$

$$\therefore x=\frac{\left[L_1^{\,2}M_1^{\,1}T_1^{\,-2}\right]}{\left[L_2^{\,2}M_2^{\,1}T_2^{\,-2}\right]} \qquad =\left(\frac{L_1}{L_2}\right)^2\left(\frac{M_1}{M_2}\right)^1\left(\frac{T_1}{T_2}\right)^{-2} \quad \ldots\ldots\ldots\ldots(1.2)$$

Where suffix 1 indicates SI system and suffix 2 indicates C.G.S. system.

Writing units

$$x=\left(\frac{m}{cm}\right)^2\left(\frac{kg}{g}\right)^1\left(\frac{s}{s}\right)^{-2}$$

$$=\left(\frac{10^2\,cm}{cm}\right)^2\left(\frac{10^3\,g}{g}\right)^1 1^{-2}$$

$$=10^4\times10^3$$

$$=10^7$$

$$\therefore x=10^7$$

$$\therefore 1\text{ J}=10^7\text{ erg}$$

---

4) If length 'L' , Force 'F' and time 'T' are taken as fundamental quantities , what will be dimensional equation of mass and density?

Ans

As force, Length and time are fundamental quantities

$$\text{force } =\left[F^1\ L^0\ \ T^0\right];\ \ \text{length } =\left[F^0\ L^1\ \ T^0\right];\ \ \text{time } =\left[F^0\ L^0\ \ T^1\right]$$

Dimensions of mass:

$$\text{Force}=\text{mass}\times\text{acc}^n$$

$$\therefore \text{mass}=\frac{\text{force}}{\text{acc}^n}$$

$$\therefore\ \text{mass }=\frac{\text{force}}{\left[\text{acc}^n\right]}$$

$$\therefore\ \text{mass }=\frac{\left[F^1\right]}{\left[L^1\ \ T^{-2}\right]}$$

$$\therefore\ \text{mass }=\left[F^1\ L^{-1}\ T^{-2}\right]$$

Dimensions of density:

$$\text{density}=\frac{\text{mass}}{\text{volume}}$$

$$\therefore\ \text{density }=\frac{\left[F^1\ \ L^{-1}\ \ T^2\right]}{\left[L^3\right]}$$

$$\therefore\ \text{density }=\left[F^1\ \ L^{-4}\ \ T^2\right]$$

---

5) If hydrostatic pressure 'P' of a liquid column depends upon the density 'd', height 'h' of liquid column and acceleration 'g' due to gravity , derive formula for pressure using dimensional analysis

Ans : Dimensions of terms involved

$$\text{pressure } = \left[L^{-1}\ M^{1}\ T^{-2}\right]$$

$$\text{density } = \left[L^{-3}\ M^{1}\ T^{0}\right]$$

$$\text{height } = \left[L^{1}\ M^{0}\ T^{0}\right]$$

$$\text{gravitational acc } = \left[L^{1}\ M^{0}\ T^{-2}\right]$$

Let

$P = k\,h^{x}d^{y}g^{z}$ **……………………….(1)**

Where k is dimensionless constant

Writing dimensions

$$P = h^{x}\ d^{y}\ g^{z}$$

$$\therefore \left[L^{-1}\ M^{1}\ T^{-2}\right] = \left[L^{1}\right]^{x}\left[L^{-3}M^{1}\right]^{y}\left[L^{1}\ T^{-2}\right]^{z}$$

$$\therefore \left[L^{-1}\ M^{1}\ T^{-2}\right] = \left[L^{x}\right]\left[L^{-3y}M^{y}\right]\left[L^{z}\ T^{-2z}\right]$$

$$\therefore \left[L^{-1}\ M^{1}\ T^{-2}\right] = \left[L^{x-3y+z}\ M^{y}\ T^{-2z}\right]$$

Equating powers on both sides

$y = 1$;

$-2z = -2$

$\therefore z = 1$

$x - 3y + z = -1$

$\therefore x - 3(1) + 1 = -1$

$\therefore x - 2 = -1$

$\therefore x = 1$

Putting values of x,y,z in equation 1

$P = k\,h\,d\,g$

Value of k has to be determined expermntly.

(We will learn in cha 5 that k =1 hence P =h d g)

---

6) If displacement of particle is given by s = at +bv^{2} where t is time and v is velocity find dimensions of a and b.

According to law of homogeneity of dimensions, each term should have same dimensions,

$$\therefore\ s = at \quad \therefore\ a = \frac{s}{t} \quad \therefore\ a = \frac{\left[L^1\right]}{\left[T^1\right]} \quad \therefore\ a = \left[L^1T^{-1}\right]$$

Also

$$\therefore\ s = \left[bv^2\right] \ \therefore\ b = \frac{s}{\left[v^2\right]} \ \therefore\ b = \frac{\left[L^1\right]}{\left[L^1T^{-1}\right]^2} \ \therefore\ b = \frac{\left[L^1\right]}{\left[L^2T^{-2}\right]} \ \therefore\ b = \left[L^{-1}T^2\right]$$

---

7) An object was weighted by physical balance and following readings were obtained: 5.04 g , 5.06 g , 4.97 g , 5.00 g, 5.06 g. find (a) Mean Value (b) Absolute error (c) % error

$$\text{Mean value} = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5} = \frac{5.04 + 5.06 + 4.97 + 5.00 + 4.93}{5}$$

= 5.00 g

Absolute error in each reading

$$\left|\Delta a_1\right| = \left|a_m - a_1\right| = 0.04\text{g}$$
$$\left|\Delta a_2\right| = \left|a_m - a_2\right| = 0.06\text{g}$$
$$\left|\Delta a_3\right| = \left|a_m - a_3\right| = 0.03\text{g}$$
$$\left|\Delta a_4\right| = \left|a_m - a_4\right| = 0.00\text{g}$$
$$\left|\Delta a_5\right| = \left|a_m - a_5\right| = 0.07\text{g}$$

Mean absolute error

$$\left|\Delta a_m\right| = \frac{\left|\Delta a_1\right| + \left|\Delta a_2\right| + \left|\Delta a_3\right| + \left|\Delta a_4\right| + \left|\Delta a_5\right|}{5}$$

$$\therefore \left|\Delta a_m\right| = \frac{0.04 + 0.06 + 0.03 + 0.00 + 0.07}{5}$$

$$\therefore \left|\Delta a_m\right| = \frac{0.20}{5}$$

$$\therefore \left|\Delta a_m\right| = 0.04\,\text{g}$$

% error

$$= \frac{\left|\Delta a_m\right|}{a_m} \times 100$$

$$= \frac{0.04}{5.00} \times 100$$

= 0.8 %

---

8) The diameter of a wire measured with a vernier calipers with least count of 0.1 mm is 3.12 cm. Find the percentage error in the measurement.

Solution :

The relative error in the measurement $= \dfrac{0.01}{3.12}$

Hence, the percentage error is given by $\dfrac{0.01}{3.12} \times 100 = 032\%$

9) **Find the percentage error in the volume V of a block of length 10 cm, breadth 20 cm and height 25 cm if the measurement is carried out with a meter scale.**

Solution:

The least count of a meter scale is 1 mm. Hence absolute error in length, breadth and height is $|\Delta l| = |\Delta b| = |\Delta h|$=1mm=0.1cm

As $V = l \times b \times h$

$\%$ error in volume $= \%$ error in $l + \%$ error in $b + \%$ error in h

$$\therefore \frac{|\Delta V|}{V} \times 100 = \frac{|\Delta l|}{l} \times 100 + \frac{|\Delta b|}{b} \times 100 + \frac{|\Delta h|}{h} \times 100$$

$$= \frac{0.1}{10} \times 100 + \frac{0.1}{20} \times 100 + \frac{0.1}{25} \times 100 = 1.9\%$$

$$\therefore \text{Percentage error} = \frac{\Delta V}{V} \times 100 = 1.9\%$$

10) **In an experiment to find the density of a solid, the mass and volume of the solid were found to be** 400.3 ± 0.02 g and 75.6 ± 0.01 cm^3 respectively. Find the relative error and percentage error in the determination of its density.

Solution:

From the given values,

$$\frac{\Delta M}{M} = \frac{0.02}{400.3} = 0.00005 \text{ and } \frac{\Delta V}{V} = \frac{0.01}{75.6} = 0.00013$$

Relation between mass, volume and density is $\rho = \frac{M}{V}$

Relative error in density = relative error in mass + relative error in volume

$$\therefore \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V} \quad \text{Neglecting sign}$$

$$\therefore \frac{\Delta \rho}{\rho} = 0.00005 + 0.00013 = 0.00018$$

Hence the percentage error in the determination of the density of the solid is

$$\frac{\Delta \rho}{\rho} \times 100\% = 0.00018 \times 100 \ \% = 0.018\%$$

Problems for practice.

1) Deduce the dimensional formula for the following quantities from their definitions :
   a. Momentum
   b. Moment of force
   c. Work
   d. Power — Ans : a. MLT^{-1} b. ML^2T^{-2} c. ML^2T^{-2} d. ML^2T^{-3}

2) Density of water is one g per cc in the C.G.S. units. Express it in S. I. units. (1000 kg/m^3)

3) The distance s covered by a body in time t is given by the relation $s = a + bt + ct^2$. What are the dimensions of a, b and c? $[L];\ [LT^{-1}];\ [LT^{-2}]$

4) Assuming that the period T of oscillation of a simple pendulum depends upon its length l and acceleration due to gravity g at the place, show that $T \propto \sqrt{\frac{l}{g}}$

5) The frequency n of vibration of a string of length l under tension F depends upon l, F and m where m is the mass per unit length of string. Show that $n \propto \frac{1}{l}\sqrt{\frac{F}{m}}$

6) In the following set of numbers, write down the number of significant digits.

   a) 0.043 b) 4.03×10^8 c) 0.0730 d) 6.00 e) 3.720 kg

   Ans : [a) 2, b) 3, c) 3, d) 3, e) 4.]

7) Round off the following numbers to three significant digits.

   a) 0.02739 km b) $7.075\times10^9 s$

   Ans : $0.0274km;\ 7.08\times10^9 s$

8) Round of the following numbers as indicated.

   a) 16.315 to 4 digits

   b) 7.998×10^5 to3 digits

   c) 0.6975 to 3 digits.

   d) 7.5936×10^{-3} to 4 digits

   e) 15.85 to 3 digits

   f) 123456 to 5 digits

   Ans : [a) 16.32 b) 7.00×10^5 c) 0.698 d) 7.594×10^{-3} e) 15.8 f) 1.2346×10^5]

9) Write down the following results to the correct number of significant digits.

   a. $0.0173+0.000070$

   b. $6.53+16.1384$ Ans : a) 0.0173 c) 22.668

10) Write the results of the following operations to the correct number of significant digits.

    a) 3.1×12.126 Ans : a) 38

11) Calculate : a) $89.54\div2.54$ b) $16.48\div0.412$ Ans : a) 35.2 b) 40.0

12) The radius of the earth is 6400 km and its mass is $5.98\times10^{24} kg$. What is the density of the earth? Ans : $5.4\times10^6 kg/m^3$

13) Considering the proton as a sphere of radius 1.35×10^{-15} m and of mass 1.67×10^{-27} kg, calculate the order of magnitude of its density. Ans : $10^{17}\,kg/m^3$

14) The radius R of a nucleus of mass number A is given by $R = 1.4\times10^{-15}A^{1/3}$. Find the order of magnitude of the radius for a nucleus with A = 235. Ans : 10^{-14} m

15) Measurement of the length of a table with a meter scale gave the following readings: 1.32 m; 1.324 m; 1.317 m; 1.321 m and 1.319 m. Write the length of the table with the uncertainty in the measurement. Ans : 1.320 ± 0.002m

16) In an experiment to find the refractive index μ of the material of a prism, the following results were obtained: 1) 1.53 2) 1.55 3) 1.54 4) 1.50 5) 1.53 6) 1.54 Calculate a) Average value of μ b) mean error c) fractional error d) percentage error.
Ans : [a) 1.53 b) 0.01 c) ±0.0065 d) $\pm0.65\%$].

17) The percentage error in the measurement of the radius r of a sphere is 0.1%. What is the percentage error introduced in the measurement of volume? Ans : (0.3%)

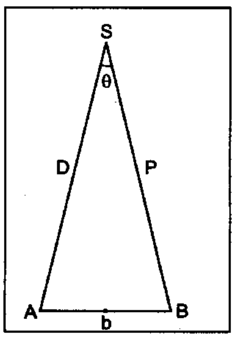
18) The following readings were obtained in an experiment to find the volume of a block with rectangular faces. Lengths = 16.03 $\pm$ 0.03 cm, breath = 12.06 $\pm$ 0.02 cm and height = 3.72 $\pm$ 0.01 cm. What is the error in volume expressed as a percentage? Ans : (0.62%)

19) In an experiment on Ohm's law, $V = 50 \pm 2.3$ volt and $I = 20 \pm 1.1$ ampere. Assume Ohm's law V = IR and calculate the percentage error in R. Ans : ($\pm$ 10.1%)

Some Interesting methods for measurements used in physics

Measurement of Length

- **Direct Methods** : a meter scale is used for lengths from 10^{-3}m to 10^{2} m. Vernier caliper is used to measure length to an accuracy of 10^{-4} m A screw gauge can be used to measure lengths as small as 10^{-5} m.
- **Parallax method:** Large distances such as the distance of a planet or a star from the earth can be measured using **parallax method.**
- To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance AB = b at the same time as shown in figure.



- We measure the angle between the two directions, along which the planet is viewed at these two points. The $\angle ASB$ represented by symbol θ is called the parallax angle or parallactic angle.
- As the plant is very far away, $\frac{b}{D} << 1$, and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius,
  AS = BS =D
  b= D θ where θ is in radians.
  $\therefore D = \frac{b}{\theta}$
- A similar method to determine the size or angular diameter of the planet. If d is the diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have $\alpha = d/D$
- The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Sine D is known, the diameter d of the planet can be determined using $\alpha = d/D$

Measurement of Mass:

- Mass is a fundamental property of matter. It does not depend on the temperature, pressure or location of the object in space. The SI unit of mass is kilogram (kg).
- But while dealing with atoms and molecules, the kilogram is an inconvenient unit. For atomic levels unified atomic mass is used. 1 unified atomic mass unit $= lu = 0.8333\times10^{-1}$ of the mass of an atom of carbon 12 in kg.
  Mass of Isotope $^{12}_{6}C$ including the mass of electrons $= 1.66\times10^{-27}$ kg
- Mass of commonly available objects can be determined by a common balance like the one used in a grocery shop.
- Large masses in the universe like planets, stars, etc, are measured using Newton's laws of gravitation.
- For measurement of small masses, we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

Measurement of Time:

- The mean solar day on the earth is considered duration of 24 hours for which an hour is of 60 minutes and each minute is of 60 seconds. A solar day is the interval from one noon to the next noon. Average of length of a solar day over a year is considered as a mean solar day.
- Atomic clock uses an atomic standard of time, which is based on the periodic vibrations produced in a cesium atom.

- In the cesium atomic clock, the second is taken as the time needed for 9,192,631,770 vibrations of the radiation corresponding to the transition between the two hyperfine states of cesium – 133 atom.
- The cesium atomic clocks are very accurate. The national standard of time interval 'second' as well as the frequency is maintained through four cesium atomic clocks. A cesium atomic clock is used at the National Physical Laboratory (NPL) New Delhi to maintain the Indian standard of time.