SAPTARSHI CLASSES PVT. LTD.

NEET/JEE

Date : 13/05/2017 **TEST ID: 120517**
Time : 02:00:00 Hrs. **PHYSICS , Chem**
Marks : 360

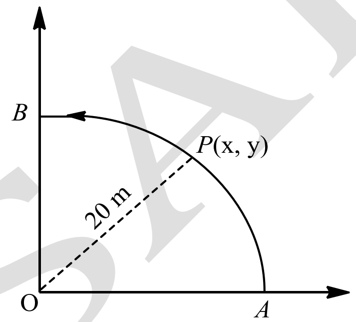
Phy : Circular Motion , Gravitation , Che : Halogen Derivatives Of Alkanes

Single Correct Answer Type

1. A bucket filled with water is tied to a rope of length 0.5 m and is rotated in a circular path in vertical pane. The least velocity it should have at the lowest point of circle so that water dose not spill is, ($g = 10$ ms^{-2})
   a) $\sqrt{5}$ ms^{-1} b) $\sqrt{10}$ ms^{-1}
   c) 5 ms^{-1} d) $2\sqrt{5}$ ms^{-1}

2. A particle describes a horizontal circle in a conical funnel whose inner surface is smooth with speed of 0.5 ms^{-1}. What is the height of the plane of circle from vertex of the funnel?
   a) 0.25 cm b) 2 cm
   c) 4 cm d) 2.5 cm

3. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. The acceleration of P when t =2s is nearly



   a) 13 ms^{-2} b) 12 ms^{-2}
   c) 7.2 ms^{-2} d) 14 ms^{-2}

4. If KE of the particle of mass m performing UCM in a circle of radius r is E. Find the acceleration of the particale
   a) $\frac{2E}{mr}$ b) $\left(\frac{2E}{mr}\right)^2$ c) $2\,Emr$ d) $\frac{4E}{mr}$

5. A particle of mass m is circulating on a circle of radius r having angular momentum L, then the centripetal force will be
   a) L^2/mr
   b) L^2m/r
   c) L^2/mr^3
   d) L^2/mr^2

6. If α is angular acceleration, ω is angular velocity and a is the centripetal acceleration then, which of the following is true?
   a) $\alpha = \frac{\omega a}{v}$ b) $\alpha = \frac{v}{\omega a}$
   c) $\alpha = \frac{va}{\omega}$ d) $\alpha = \frac{a}{\omega v}$

7. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is
   a) $\frac{ML\omega^2}{2}$ b) $ML\omega^2$
   c) $\frac{ML\omega^2}{4}$ d) $\frac{ML^2\omega^2}{2}$

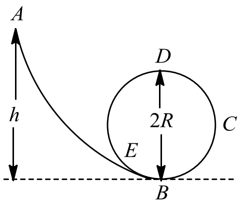
8. A car wheel is rotated to uniform angular acceleration about its axis. Initially its angular velocity is zero. It rotates through an angle θ_1 in the first 2 s , in the next 2 s, it rotates through an additional angle θ_2, the ratio of $\frac{\theta_2}{\theta_1}$ is
   a) 1 b) 2 c) 3 d) 5

9. A particle has velocity $\sqrt{3rg}$ at the highest pint in vertical circle. Find the ratio of tensions at the highest and lowest point
   a) 1 : 6 b) 1 : 4 c) 1 : 3 d) 1 : 2

10. A stone tied to a string of length L is whirled in a vertical circle, with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of change in its velocity as it reaches a position, where the string is horizontal is

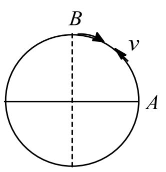
a) $\sqrt{u^2 - 2\,g\,L}$ b) $\sqrt{2\,g\,L}$
c) $\sqrt{u^2 - g\,L}$ d) $\sqrt{2(u^2 - g\,L)}$

11. A frictionless track $ABCDE$ ends in a circular loop of radius R, figure. A body slides down the track from point A which is at a height $h =$ 5 cm. Maximum value of R for the body to successfully complete the loop is



a) 5 cm b) 15/4 cm c) 10/3 cm d) 2 cm

12. A body of mass m is moving with a uniform speed v along a circle of radius r, what is the average acceleration in going from A to B?



a) $2v^2/\pi r$ b) $2\sqrt{2}v^2/\pi r$ c) $v^2/\pi r$ d) None of these

13. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

a) $m_1 r_1 : m_2 r_2$ b) $m_1 : m_2$
c) $r_1 : r_2$ d) $1 : 1$

14. For a particle in uniform circular motion, thee acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

a) $\frac{v^2}{R}\hat{\imath} + \frac{v^2}{R}\hat{\jmath}$
b) $-\frac{v^2}{R}\cos\theta\,\hat{\imath} + \frac{v^2}{R}\sin\theta\,\hat{\jmath}$
c) $-\frac{v^2}{R}\sin\theta\,\hat{\imath} + \frac{v^2}{R}\cos\theta\,\hat{\jmath}$
d) $-\frac{v^2}{R}\cos\theta\,\hat{\imath} - \frac{v^2}{R}\sin\theta\,\hat{\jmath}$

15. A body of mass m is moving in a circle of radius r with a constant speed v. The force on the body is $\frac{mv^2}{r}$ and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle

a) $\frac{mv^2}{r} \times \pi r$ b) Zero
c) $\frac{mv^2}{r^2}$ d) $\frac{\pi r^2}{mv^2}$

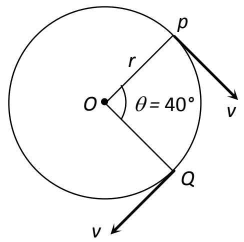
16. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of $2m/\sec^2$, What is the acceleration of the car

a) $2m/\sec^2$ b) $2.7m/\sec^2$
c) $1.8m/\sec^2$ d) $9.8m/\sec^2$

17. A1 kg stone at the end of 1 m long string is whirled in a vertical circle at constant speed of 4 m/sec. The tension in the string is 6 N, when the stone is at ($g = 10\ m/\sec^2$)

a) Top of the circle b) Bottom of the circle
c) Half way down d) None of the above

18. A particle is moving on a circular path of radius r with uniform velocity v. The change in velocity when the particle moves from P to Q is ($\angle POQ = 40°$)



a) $2v\cos 40°$ b) $2v\sin 40°$
c) $2v\sin 20°$ d) $2v\cos 20°$

19. For a body moving in a circular path, a condition for no skidding if μ is the coefficient of friction, is

a) $\frac{mv^2}{r} \le \mu mg$ b) $\frac{mv^2}{r} \ge \mu mg$
c) $\frac{v}{r} = \mu g$ d) $\frac{mv^2}{r} = \mu mg$

20.

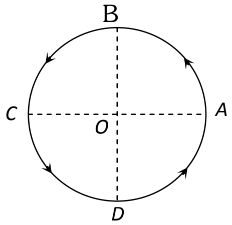


Figure shows a body of mass m moving with a uniform speed v along a circle of radius r. The change in velocity in going from A to B is

a) $v\sqrt{2}$ b) $v/\sqrt{2}$ c) v d) zero

21. A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per $minute$. The total tension in the string when the stone is at its lowest point is

a) mg

b) $m(g + \pi n r^2)$

c) $m(g + \pi n r)$

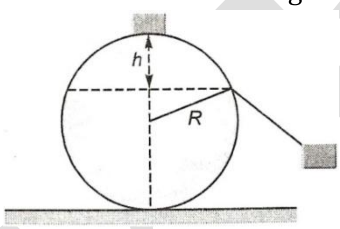
d) $m\{g + (\pi^2\, n^2\, r)/900\}$

22. If a_r and a_t represent radial and tangential accelerations, the motion of a particle will be uniformly circular if

a) $a_r = 0$ and $a_t = 0$ b) $a_r = 0$ but $a_t \neq 0$

c) $a_r \neq 0$ but $a_t = 0$ d) $a_r \neq 0$ and $a_t \neq 0$

23. A particle originally at a rest at the highest point of a smooth circle in a vertical plane, is gently pushed and starts sliding along the circle. It will leave the circle at a vertical distance h below the highest point such that



a) $h = 2R$ b) $h = \frac{R}{2}$

c) $h = R$ d) $h = \frac{R}{3}$

24. A cyclist goes round a circular path of circumference 34.3 m in $\sqrt{22}$ s, the angle made by him with the vertical will be

a) 45° b) 40° c) 42° d) 48°

25. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of$a$. What will be the angle of inclination with vertical?

a) $\tan^{-1}\left(\frac{a}{g}\right)$

b) $\tan^{-1}\left(\frac{g}{a}\right)$

c) $\text{cas}^{-1}\left(\frac{a}{g}\right)$

d) $\text{cas}^{-1}\left(\frac{g}{a}\right)$

26. A stone of mass 1 kg tied to the end of a string of length 1 m, is whirled in a horizontal circle with a uniform angular velocity 2 rads^{-1}. The tension of the string is (in newton)

a) 2 b) $\frac{1}{3}$ c) 4 d) $\frac{1}{4}$

27. The ratio of the angular speed of minutes hand and hour hand of a watch is

a) 6 : 1 b) 12 : 1

c) 1 : 6 d) 1 : 12

28. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time as $a_c = k^2 r t^4$, where k is a constant. The power delivered to the particle by the forces acting on its is

a) Zero b) $mk^2r^2t^2$

c) $\frac{1}{3}mk^2r^2t^2$ d) $2mk^2r^2t^3$

29. A sphere is suspended by a thread of length l. The minimum horizontal velocity which has to be imparted to the sphere for it to reach the height of suspension is

a) $2\sqrt{gR}$ b) $\sqrt{2gl}$

c) 2 g l d) g l

30. An aeroplane flying at a velocity of 900 kmh^{-1} loops the loop. If the maximum force pressing the pilot against the seat is five times its weight, the loop radius should be

a) 1594 m

b) 1402 m

c) 1315 m

d) 1167 m

31. Three identical bodies of mass M are located at the vertices of an equilateral triangle of side L. They revolve under the effect of mutual

gravitational force in a circular orbit, circumscribing the triangle while preserving the equilateral triangle. Their orbital velocity is

a) $\sqrt{\frac{GM}{L}}$ b) $\sqrt{\frac{3GM}{2L}}$ c) $\sqrt{\frac{3GM}{L}}$ d) $\sqrt{\frac{2GM}{3L}}$

32. Two balls, each of radius R, equal mass and density are placed in contact, then the force of gravitation between them is proportional to

a) $F \propto \frac{1}{R^2}$ b) $F \propto R$

c) $F \propto R^4$ d) $F \propto \frac{1}{R}$

33. The density of earth in terms of acceleration due to gravity (g), radius of earth (R) and universal gravitational constant (G) is

a) $\frac{4\pi RG}{3g}$ b) $\frac{3\pi RG}{4g}$ c) $\frac{4g}{3\pi RG}$ d) $\frac{3g}{4\pi RG}$

34. A body is taken to a height of nR from the surface of the earth. The ratio of the acceleration due to gravity on the surface to that at the altitude is

a) $(n+1)^2$ b) $(n+1)^{-2}$

c) $(n+1)^{-1}$ d) $(n+1)$

35. A satellite is orbiting around the earth. By what percentage should we increase its velocity, so as to enable it escape away from the earth?

a) 41.4% b) 50% c) 82.8% d) 100%

36. The mass of a planet is six times that of the earth. The radius of the planet is twice that of the earth. If the escape velocity from the earth is v, then the escape velocity from the planet is

a) $\sqrt{3}v$ b) $\sqrt{2}v$ c) v d) $\sqrt{5}v$

37. If an object of mass m is taken from the surface of earth (radius R) to a height $2R$, then the work done is

a) $2mgR$ b) mgR c) $\frac{2}{3}mgR$ d) $\frac{3}{2}mgR$

38. The gravitational potential energy of a body of mass m at a distance r from the centre of the earth is U. What is the weight of the body at this distance?

a) U b) Ur c) $\frac{U}{r}$ d) $\frac{U}{2r}$

39. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

a) $-\frac{4Gm}{r}$ b) $-\frac{6Gm}{r}$

c) $-\frac{9Gm}{r}$ d) zero

40. The distance of a planet from the sun is 5 times, the distance between the earth and the sun. the time period of the planet is

a) $6^{3/2}T$ yr b) $5^{3/2}T$ yr

c) $5^{3/1}T$ yr d) $5^{1/2}T$ yr

41. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

a) $R^{\left(\frac{n+1}{2}\right)}$ b) $R^{\left(\frac{n-1}{2}\right)}$ c) R^n d) $R^{\left(\frac{n-2}{2}\right)}$

42. In a satellite, if the time of revolution is T, then KE is proportional to

a) $\frac{1}{T}$ b) $\frac{1}{T^2}$ c) $\frac{1}{T^3}$ d) $T^{-2/3}$

43. If ρ is the density of the planet, the time period of nearby satellite is given by

a) $\sqrt{\frac{4\pi}{3G\rho}}$ b) $\sqrt{\frac{4\pi}{G\rho}}$ c) $\sqrt{\frac{3\pi}{G\rho}}$ d) $\sqrt{\frac{\pi}{G\rho}}$

44. The acceleration due to gravity is g at a point distant r from the centre of earth of radius R. If $r < R$, then

a) $g \propto r$ b) $g \propto r^2$

c) $g \propto r^{-1}$ d) $g \propto r^{-2}$

45. A spherical planet has a mass M_P and diameter D_P. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity, equal to

a) $4GM_P/D_P^2$ b) GM_Pm/D_P^2

c) GM_P/D_P^2 d) $4GM_Pm/D_P^2$

46. The depth d at which the value of acceleration due to gravity becomes $1/n$ times the value of

the surface, is [R = radius of the earth]

a) $\frac{R}{n}$ b) $R\left(\frac{n-1}{n}\right)$ c) $\frac{R}{n^2}$ d) $R\left(\frac{n}{n+1}\right)$

47. The angular velocity of rotation of star (of mass M and radius R) at which the matter start to escape from its equator will be

a) $\sqrt{\frac{2GM^2}{R}}$ b) $\sqrt{\frac{2GM}{g}}$ c) $\sqrt{\frac{2GM}{R^3}}$ d) $\sqrt{\frac{2GR}{M}}$

48. If the radius of a planet is R and its density is ρ, the escape velocity from its surface will be

a) $v_e \propto \rho R$ b) $v_e \propto \sqrt{\rho R}$

c) $v_e \propto \frac{\sqrt{\rho}}{R}$ d) $v_e \propto \frac{1}{\sqrt{\rho R}}$

49. Four particles each of mass M, are located at the vertices of a square with side L. The gravitational potential due to this at the centre of the square is

a) $-\sqrt{32}\frac{GM}{L}$ b) $-\sqrt{64}\frac{GM}{L^2}$ c) Zero d) $\sqrt{32}\frac{GM}{L}$

50. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are distance d apart. The minimum velocity with which a particle of mass m shoule be projected from a point midway between their centres so that it escapes to infinity is

a) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ b) $2\sqrt{\frac{2G}{d}(M_1+M_2)}$

c) $2\sqrt{\frac{Gm}{d}(M_1+M_2)}$ d) $2\sqrt{\frac{Gm(M_1+M_2)}{d(R_1+R_2)}}$

51. The ratio of the radii of planets A and B is k_1 and ratio of acceleration due to gravity on them is k_2. The ratio of escape velocities from them will be

a) k_1k_2 b) $\sqrt{k_1k_2}$ c) $\sqrt{\frac{k_1}{k_2}}$ d) $\sqrt{\frac{k_2}{k_1}}$

52. The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M, to transfer it from a circular orbit of radius R_1 to another of radius $R_2 (R_2 > R_1)$ is

a) $GmM\left(\frac{1}{R_1^2}-\frac{1}{R_2^2}\right)$ b) $GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

c) $2GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ d) $\frac{1}{2}GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

53. A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k < 1$). If resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be : (R = radius of earth)

a) $\frac{R}{k^2+1}$ b) $\frac{R}{k^2-1}$ c) $\frac{R}{1-k^2}$ d) $\frac{R}{k+1}$

54. The condition for a uniform spherical mass m of radius r to be a black hole is [G = gravitational constant and g = acceleration due to gravity]

a) $(2Gm/r)^{1/2} \le c$ b) $(2Gm/r)^{1/2} = c$

c) $(2Gm/r)^{1/2} \ge c$ d) $(gm/r)^{1/2} \ge c$

55. A satellite A of mass m is at a distance of r from the centre of the earth. Another satellite B of mass $2m$ is at a distance of $2r$ from the earth's centre. Their time periods are in the ratio of

a) 1 : 2 b) 1 : 16 c) 1 : 32 d) $1 : 2\sqrt{2}$

56. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively. Then the ratio $\frac{v_1}{v_2}$ is

a) $\frac{r_1}{r_2}$ b) $\left(\frac{r_1}{r_2}\right)^2$ c) $\frac{r_2}{r_1}$ d) $\left(\frac{r_2}{r_1}\right)^2$

57. If mass of earth is M, radius is R and gravitational constant is G, then work done to take 1 kg mass from earth surface to infinity will be

a) $\sqrt{\frac{GM}{2R}}$ b) $\frac{GM}{R}$ c) $\sqrt{\frac{2GM}{R}}$ d) $\frac{GM}{2R}$

58. Two small and heavy spheres, each of mass M, are placed a distance r apart on a horizontal surface. The gravitational potential at the mid-point on the line joining the centre of the spheres is

a) Zero b) $-\frac{GM}{r}$ c) $-\frac{2GM}{r}$ d) $-\frac{4GM}{r}$

59. A particle falls towards earth from infinity. It's velocity on reaching the earth would be
a) Infinity b) $\sqrt{2gR}$ c) $2\sqrt{gR}$ d) Zero

60. If the angular speed of the earth is doubled, the value of acceleration due to gravity (g) at the north pole
a) Doubles b) Becomes half
c) Remains same d) Becomes zero

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: ANSWER KEY :

1)	c	2)	d	3)	d	4)	a
5)	c	6)	a	7)	a	8)	c
9)	b	10)	d	11)	d	12)	b
13)	c	14)	d	15)	b	16)	b
17)	a	18)	c	19)	a	20)	a
21)	d	22)	c	23)	d	24)	a
25)	a	26)	c	27)	b	28)	d
29)	b	30)	a	31)	a	32)	c
33)	d	34)	a	35)	a	36)	a
37)	c	38)	c	39)	c	40)	b
41)	a	42)	d	43)	c	44)	a
45)	a	46)	b	47)	c	48)	b
49)	a	50)	a	51)	b	52)	d
53)	c	54)	c	55)	d	56)	c
57)	b	58)	d	59)	b	60)	c

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: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(c)**

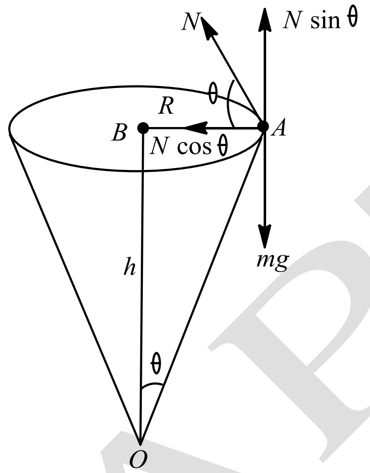
For water not to spill out of the bucket,

$v_{\min} = \sqrt{5gR}$ (at the lowest point)

$= \sqrt{5 \times 10 \times 0.5} = 5\ \text{ms}^{-1}$

2 **(d)**

The various forces acting on the particle, are its weight mg acting vertically downwards, normal reaction N. Equating the vertical forces, we have



$N \sin\theta = mg$ ... (i)

Also, centripetal force,

$\frac{mv^2}{R} = N\cos\theta$ .... (ii)

From Eqs. (i) and (ii), we get

$\tan\theta = \frac{Rg}{v^2}$ .... (iii)

Also, from triangle OAB,

$\tan\theta = \frac{R}{h}$ ..... (iv)

Equating Eqs. (iii) and (iv), we get

$h = \frac{v^2}{g}$

Given, $v = 0.5\ \text{ms}^{-1}$ and $g = 10\ \text{ms}^{-2}$

$\therefore\ h \frac{(0.5)^2}{10} = 0.025\ \text{m} = 2.5\ \text{cm}$

3 **(d)**

Given, $s = t^3 + 5$

Speed, $v = \frac{ds}{dt} = 3t^2$

and rate of change of speed, $a_t = \frac{dv}{dt} = 6t$

$\therefore$ Tangential acceleration at $t = 2\ s$,

$a_t = 6 \times 2 = 12\ \text{ms}^{-2}$

and at $t = 2s, v = 3(2)^2 = 12\text{ms}^{-1}$

$\therefore$ Centripetal acceleration, $a_c = \frac{v^2}{R} = \frac{144}{20}\text{ms}^{-2}$

$\therefore$ Net acceleration $= a_t^2 + a_i^2 \approx 14\text{ms}^{-2}$

4 **(a)**

Kinetic energy

$E = \frac{1}{2}mv^2$

or $\frac{1}{2}mr\frac{v^2}{r} = E$

or $\frac{1}{2}mra = E$

or $a = \frac{2E}{mr}$

5 **(c)**

Angular momentum $L = r \times p = r \times m \times v$

$v = \frac{L}{mr}$ .... (i)

Now, as centripetal force, $F_c = \frac{mv^2}{r}$ .... (ii)

Substituting the value of v from Eq. (i) in Eq. (ii), we get

$$F_c = \frac{m}{r}\left[\frac{L}{mr}\right]^2 = \frac{L^2}{mr^3}$$

6 **(a)**

Centripetal acceleration

$a = \frac{v}{t}$

Angular acceleration $\propto = \frac{\omega}{t} = \frac{\omega v}{vt}$

$\therefore \propto = \frac{\omega\, a}{v}$

7 **(a)**

The centre of gravity of other tube will be at length $L/2$

So radius $r = L/2$

Centripetal force, $= M\, r\, \omega^2 = M(L/2)\omega^2 = ML\omega^2/2$

8 **(c)**

$\alpha = \frac{\omega}{t}$ and $\omega = \frac{\theta}{t}$

$\therefore \ \alpha = \frac{\theta}{t^2}$

But $\alpha =$constant

So, $\frac{\theta_1}{\theta_1+\theta_2} = \frac{(2)^2}{(2+2)^2}$

or $\frac{\theta_1}{\theta_1+\theta_2} = \frac{1}{4}$

or $\frac{\theta_1+\theta_2}{\theta_1} = \frac{4}{1}$

or $1 + \frac{\theta_2}{\theta_1} = \frac{4}{1}$

$\therefore \ \frac{\theta_2}{\theta_1} = 3$

9 **(b)**

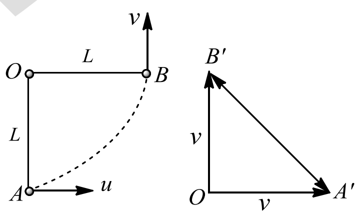
$$T_{\text{top}} = \frac{mv^2}{r} - mg = 2mg$$

$$\frac{T_{\text{top}}}{T_{\text{bottom}}} = \frac{2mg}{2mg + 6mg} = \frac{1}{4}$$

10 **(d)**

The velocity at B is v, where

$v^2 = u^2 - 2\text{g}\, L$, in vertically upward direction. As is clear from figure change in velocity



$\overrightarrow{\mathbf{OB}}'' = \overrightarrow{\mathbf{OA}} = \overrightarrow{\mathbf{A''B''}}$

$$= \sqrt{u^2 + v^2} = \sqrt{u^2 + (u^2 - 2\text{g}L)} = \sqrt{2(u^2 - \text{g}L)}$$

11 **(d)**

For successfully completing the loop,

$$h = \frac{5}{4}R \Rightarrow R = \frac{2h}{5} = \frac{2 \times 5}{5} = 2\text{cm}$$

12 **(b)**

Here, $T = \frac{2\pi r}{4v} = \frac{\pi r}{2v}$

Change in velocity is going from A to $B = v\sqrt{2}$

Average acceleration $= \frac{v\sqrt{2}}{\pi r/2v} = \frac{2\sqrt{2}v^2}{\pi r}$

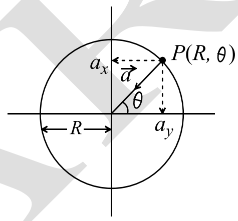
13 **(c)**

They have same ω

Centripetal acceleration $= \omega^2 r$

$$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

14 **(d)**



$$\vec{a} = -\frac{v^2}{R}\cos\theta\,\hat{\imath} - \frac{v^2}{R}\sin\theta\,\hat{\jmath}$$

15 **(b)**

Work done by centripetal force is always zero

16 **(b)**

Net acceleration in nonuniform circular motion,

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(2)^2 + \left(\frac{900}{500}\right)^2} = 2.7\ m/s^2$$

$a_t =$ tangential acceleration

$a_c =$ centripetal acceleration $= \frac{v^2}{r}$

17 **(a)**

$$mg = 1 \times 10 = 10N, \frac{mv^2}{r} = \frac{1 \times (4)^2}{1} = 16$$

Tension at the top of circle $= \frac{mv^2}{r} - mg = 6N$

Tension at the bottom of circle $= \frac{mv^2}{r} + mg = 26N$

18 **(c)**

Change in velocity $= 2v\sin(\theta/2) = 2v\sin 20°$

19 **(a)**

The value of frictional force should be equal or more than required centripetal force. i.e.

$\mu mg \geq \frac{mv^2}{r}$

20 **(a)**

$$|\overrightarrow{\Delta v}| = 2v\sin(\theta/2) = 2v\sin\left(\frac{90}{2}\right) = 2v\sin 45$$
$$= v\sqrt{2}$$

21 **(d)**

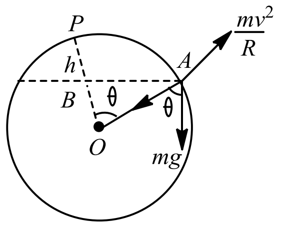
$$T = mg + m\omega^2 r = m\{g + 4\pi^2 n^2 r\}$$
$$= m\left\{g + \left(4\pi^2\left(\frac{n}{60}\right)^2 r\right)\right\} = m\left\{g + \left(\frac{\pi^2 n^2 r}{900}\right)\right\}$$

22 **(c)**

In uniform circular motion tangential acceleration remains zero but magnitude of radial acceleration remains constant.

23 **(d)**

From law of conservation of energy, potential energy of fall gets converted to kinetic energy.



$\therefore$ PE = KE

$$mgh = \frac{1}{2}mv^2$$

or $v = \sqrt{2gh}$ .... (i)

Also, the horizontal component of force is equal centrifugal force.

$$\therefore \quad mg\cos\theta = \frac{mv^2}{R} \text{ ... (ii)}$$

From Eq. (i)

$$\therefore \quad mg\cos\theta = \frac{2mgh}{R} \text{ ... (iii)}$$

From $\Delta\, AOB$,

$$\cos\theta = \frac{(R-h}{R}$$
$$\Rightarrow \quad mg\left(\frac{(R-h)}{R}\right) = \frac{2mgh}{R}$$
$$\Rightarrow \quad 3h = R$$
$$\Rightarrow h = \frac{R}{3}$$

24 **(a)**

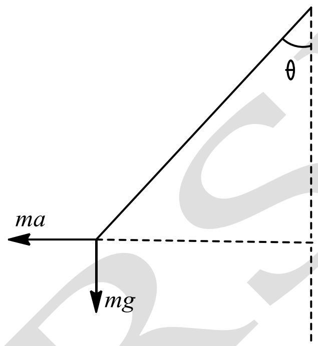
Here, $\quad 2\pi r = 34.3 \Rightarrow r = \frac{3403}{2\pi}$ and $v = \frac{2\pi r}{T}$

$$= \frac{2\pi r}{\sqrt{22}}$$

Angle of banking $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = 45°$

25 **(a)**

Let the angle from the vertical beθ. The diagram showing the different forces is given



Form the figure, $\quad \tan\theta = \frac{a}{g}$

$$\theta = \tan^{-1}\frac{a}{g}$$

26 **(c)**

The tension of the string,

$T = mr\omega^2$

$= 1 \times 1 \times (2)^2 = 4\text{N}$

27 **(b)**

Angular speed of minute hand,

$\omega_m = \frac{2\pi}{60\times 60}$ rad s^{-1}

Angular speed of hour hand,

$\omega_h = \frac{2\pi}{12\times 60\times 60}$ rad s^{-1}

$\therefore \frac{\omega_m}{\omega_h} = 12$

28 **(d)**

$a_c = k^2 r t^4 = \frac{v^2}{r}$ or $v = krt^2$

The tangential acceleration is $a_T = \frac{dv}{dt} = 2krt$

The tangential force on the particle, $F_T = ma_T = 2mkrt$

Power delivered to the particle

$$= F_T = ma_T = 2mkrt = F_T v = (2mkrt)(krt)^2$$
$$= 2mk^2r^2t^3$$

29 **(b)**

To reach the height of suspension, $h = l$

$v = \sqrt{2gh} = \sqrt{2gl}$

30 **(a)**

Here, $v = 900 \text{ km h}^{-1}$

$= \dfrac{900 \times 1000}{60 \times 60} \text{ms}^{-1} = 250 \text{ ms}^{-1}$

Minimum force is at the bottom of the vertical circle

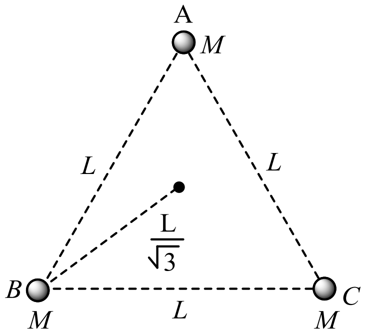
$F_{\max} = \dfrac{mv^2}{r} + mg = 5\, mg$

$\therefore\ v^2 = 4\, gr$

or $r = \frac{v^2}{4g} = \frac{250\times250}{4\times980} = 1594$ m

31 **(a)**

Given, $F_1 = F_2 = F$ and $\theta = 60°$



Resultant force $= \sqrt{3}\, F$

$\therefore$ Force on mass at A due to mass at B and C

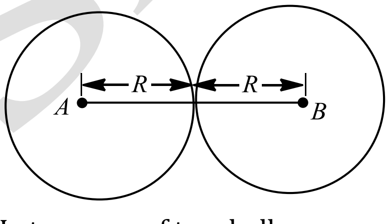
$$= \sqrt{3}\left(\frac{GM^2}{L^2}\right)$$

Centripetal force for circumscribing the triangle in a circular orbit is provided by mutual gravitational interaction.

ie, $\dfrac{Mv^2}{(L/\sqrt{3})} = \sqrt{3}\left(\dfrac{GM^2}{L^2}\right)$

or $v = \sqrt{\dfrac{GM}{L}}$

32 **(c)**



Let masses of two balls are $m_1 = m_2 = m$ (given) and the density be ρ.

Distance between their centres $= AB = 2R$

Thus, the magnitude of the gravitational force F that two balls separated by a distance $2R$ exert on each other is

$$F = G\frac{(m)(m)}{(2R)^2}$$

$$= G\frac{m^2}{4R^2} = G\frac{\left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$$

$\therefore \qquad F \propto R^4$

33 **(d)**

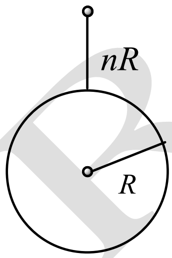
Acceleration due to gravity

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$$

$\therefore \qquad \rho = \dfrac{3g}{4\pi GR}$

34 **(a)**

Acceleration due to gravity at a height above the earth surface



$$g'' = g\left(\frac{R}{R+h}\right)^2$$

$$\frac{g}{g''} = \left(\frac{R+h}{R}\right)^2$$

$$\frac{g}{g''} = \left(\frac{R+nR}{R}\right)^2$$

$$\frac{g}{g''} = (1+n)^2$$

35 **(a)**

The velocity with which satellite is orbiting around the earth is the orbital velocity (v_o) and that required to escape out of gravitational pull of earth is the escape velocity (v_e).

We know that

$$v_e = \sqrt{2gR} \text{ and } v_o = \sqrt{gR}$$

$\therefore$ Increase in velocity required

$$= \frac{v_e - v_o}{v_o} = \frac{\sqrt{2gR} - \sqrt{gR}}{\sqrt{gR}}$$

$$= \sqrt{2} - 1 = 0.414$$

Percent increase in velocity required

$= 0.414 \times 100 = 41.4\%$

36 **(a)**

Escape velocity, $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$

Given, $\quad M_p = 6M_e, R_p = 2R_e$

$\therefore \qquad v_p = \sqrt{\dfrac{2G \cdot 6M_e}{(2R_e)}} = \sqrt{3}\, v_e$

37 **(c)**
Work done

$$W = \Delta U = \frac{mgh}{1 + \frac{-h}{R}}$$

Substituting $R = \dfrac{h}{L}$ we get

$$\Delta U = \frac{mg \times 2R}{1 + 2}$$

$$\Delta U = \frac{2mgR}{3}$$

38 **(c)**

Gravitational potential energy, $U = \dfrac{GMm}{r}$

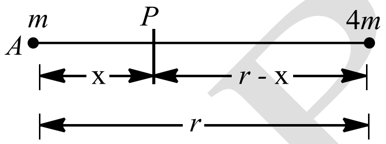
or $\qquad U = \dfrac{GMm}{r^2} \times r$

or $\qquad U = g \times mr$

or $\qquad U = (mg)r$

or $\qquad mg = \dfrac{U}{r}$

39 **(c)**
Let gravitation field is zero at P as shown in figure.



$\therefore \qquad \dfrac{Gm}{x^2} = \dfrac{G(4m)}{(r-x)^2}$

$\Rightarrow \qquad 4x^2 = (r-x)^2$

$\Rightarrow \qquad 2x = r - x$

$\Rightarrow \qquad x = \dfrac{r}{3}$

$\therefore \qquad V_p = \dfrac{Gm}{x} - \dfrac{G(4m)}{r-x}$

$$= -\frac{3Gm}{r} - \frac{6Gm}{r} = -\frac{9Gm}{r}$$

40 **(b)**
From Kepler's third law of planetary motion

$$T^2 \propto R^3$$

Given, $\quad R_1 = R, \qquad R_2 = 5R$

$\therefore \qquad \dfrac{T_1^2}{T_2^2} = \dfrac{R^3}{(5R)^3}$

$\Rightarrow \qquad \dfrac{T_1}{T_2} = \dfrac{1}{(5)^{3/2}}$

$\qquad T_2 = 5^{3/2}T_1$

$\therefore \qquad T_2 = 5^{\frac{3}{2}}T \qquad\qquad [\because T_1 = T]$

41 **(a)**
The necessary centripetal force required for a planet to move round the sun = gravitational force exerted on it

ie, $\qquad \dfrac{mv^2}{R} = \dfrac{GM_em}{R^n}$

$$v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$$

Now, $\quad T = \dfrac{2\pi R}{v} = 2\pi R \times \left(\dfrac{R^{n-1}}{GM_e}\right)^{1/2}$

$\Rightarrow \qquad = 2\pi\left(\dfrac{R^2 \times R^{n-1}}{GM_e}\right)^{1/2}$

$\qquad = 2\pi\left(\dfrac{R^{(n+1)/2}}{(GM_e)^{1/2}}\right)$

$\Rightarrow \qquad T \propto R^{(n+1)/2}$

42 **(d)**

Velocity of satellite $v = \sqrt{\dfrac{GM}{r}}$

$\qquad \text{KE} \propto v^2 \propto \dfrac{1}{r}$

and $\qquad T^2 \propto r^3$

$\qquad \text{KE} \propto T^{-2/3}$

43 **(c)**
Time period of nearby satellite

$$T = 2n\sqrt{\frac{r^3}{GM}}$$

$$= 2\pi\sqrt{\frac{R^3}{GM}}$$

$$= \frac{2\pi(R^3)^{1/2}}{\left[G\, \frac{4}{3}\pi R^3 \rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

44 **(a)**
Inside the earth $g' = \frac{4}{3}\pi\rho G r \;\; \therefore g' \propto r$

45 **(a)**
Gravitational attraction force on particle B

$F_g = \frac{GM_P m}{(D_P/2)^2}$

Acceleration of particle due to gravity

$a = \frac{F_g}{m} = \frac{4GM_P}{D_P^2}$

46 **(b)**

$$g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \left(\frac{n-1}{n}\right)R$$

47 **(c)**

Escape velocity $v = \sqrt{\frac{2GM}{R}}$

If star rotates with angular velocity ω

Then $\omega = \frac{v}{R} = \frac{1}{R}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2GM}{R^3}}$

48 **(b)**

$$v_e = R\sqrt{\frac{8}{3}G\pi\rho} \quad \therefore v_e \propto R\sqrt{\rho}$$

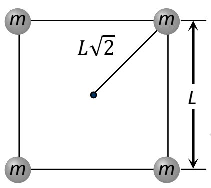
49 **(a)**

Potential at the centre due to single mass $= \frac{-GM}{L/\sqrt{2}}$

Potential at the centre due to all four masses

$$= -4\frac{GM}{L/\sqrt{2}} = -4\sqrt{2}\frac{GM}{L}$$

$$= -\sqrt{32} \times \frac{GM}{L}$$



50 **(a)**

Gravitational potential at mid point

$$V = \frac{-GM_1}{d/2} + \frac{-GM_2}{d/2}$$

Now, $PE = m \times V = \frac{-2Gm}{d}(M_1 + M_2)$

(m = mass of particle)

So, for projecting particle from mid point to infinity

$KE = |PE|$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{2\,Gm}{d}(M_1 + M_2) \Rightarrow v = 2\sqrt{\frac{G(M_1 + M_2)}{d}}$$

51 **(b)**

$$v = \sqrt{2gR} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{g_A}{g_B} \times \frac{R_A}{R_B}} = \sqrt{k_1 \times k_2} = \sqrt{k_1 k_2}$$

52 **(d)**

$$-\frac{GMm}{2R_1} + KE = -\frac{GMm}{2R_2}$$

$$KE = \frac{GMm}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

53 **(c)**

Kinetic energy = Potential energy

$$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1 + \frac{h}{R}} \Rightarrow \frac{1}{2}mk^2 2gR = \frac{mgh}{1 + \frac{h}{R}} \Rightarrow h = \frac{Rk^2}{1 - k^2}$$

Height of Projectile from the earth's surface $= h$

Height from the centre $r = R + h = R + \frac{Rk^2}{1-k^2}$

By solving $r = \frac{R}{1-k^2}$

54 **(c)**

Escape velocity for that body $v_e = \sqrt{\frac{2Gm}{r}}$

v_e should be more than or equal to speed of light

$$i.e. \sqrt{\frac{2Gm}{r}} \geq c$$

55 **(d)**

Mass of the satellite does not affect the time period

$$\frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \left(\frac{1}{8}\right)^{1/2} = \frac{1}{2\sqrt{2}}$$

56 **(c)**

$v_1 r_1 = v_2 r_2$ [$\because$ angular momentum is constant]

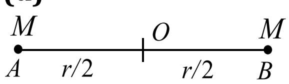
57 **(b)**

Potential energy of the 1 kg mass which is placed at the earth surface $= -\frac{GM}{R}$

Its potential energy at infinite = 0

$\therefore$ Work done = change in potential energy $= \frac{GM}{R}$

58 **(d)**



Gravitational potential of A at $O = -\frac{GM}{r/2} = -\frac{2GM}{r}$

For B, potential at $O = -\frac{GM}{r/2} = -\frac{2GM}{r}$

$\therefore$ Total potential $= -\frac{4GM}{r}$

59 **(b)**

This should be equal to escape velocity $i.e., \sqrt{2gR}$

60 **(c)**

Acceleration due to gravity at poles is independent of the angular speed of earth